OPTIMUM DESIGN OF PRESTRESSED CONCRETE GIRDER USING JAYA ALGORITHM

[a] Imran Shabbir, [b] Qaisar-uz-Zaman Khan

a: University of Engineering and Technology, Taxila, Civil Engineering Department, imran.shabbir2@students.uettaxila.edu.pk
b: University of Engineering and Technology, Taxila, Civil Engineering Department, drqaiser@uettaxila.edu.pk

Abstract- In the past few decades, there has been a significant rise in utilization of meta-heuristic algorithms, such as Genetic algorithm (GA), Ant colony optimization (ACO), Particle Swarm Optimization (PSO), Harmony Search (HS), Big Bang-Big Crunch (BB-BC), Artificial Bee Colony Algorithm (ABC) and Whale Optimization (WO), etc., for design and analysis of concrete structures. The objective of this paper is to apply one such algorithm, known as Jaya Algorithm, to optimize the design of Prestressed Concrete I Girder. Jaya Algorithm has been successfully applied in the field of structural engineering in the past. However, its robustness and efficacy for optimal design of prestressed girder is yet to be tested. For this study, a program was created on MATLAB, following the design guidelines of American Association of State Highway and Transportation Officials (AASHTO) LRFD Bridge Design Specifications for prestressed I girder design. Cross sectional dimensions and number of strands per tendon were used as variables. Flexural stress, shear stress, deflections limitations, and minimum geometric dimensions were used as constraints. To demonstrate the effectiveness of this technique, design optimization for a 40m long prestressed I Girder was successfully carried out in this study. The favorable results of this study indicate that Jaya Algorithm is highly effective and can be recommended for use in the industry.

Keywords- Jaya Algorithm, Optimization, Prestressed Post-tensioned I Girder, Meta-heuristic techniques.

1 Introduction

Prestressed Concrete Girders are used quite frequently in the highway construction for small-to-medium span bridges. Prestressing of girder reduces the overall impact of external forces by induction of internal stresses. According to National Highway Authority Pakistan (Project Completion Report BMS, 2018), almost one-third of more than four thousand bridges make use of Prestressed Concrete (PSC) Girders in their design. Of those, the most used PSC girder is the I-Girder. Typically, an iterative approach of trial and error is used for the design of structures. This iterative approach, depending upon experience of the designer, might result in high cost, time, and human effort and even then, the optimal design cannot be guaranteed. This is due to involvement of multiple variables such as cross-sectional dimensions, number of prestressing tendons, and number of strands per tendon, impacting the total cost. Thus, it is very difficult to find the desired optimal design using traditional methods due to large number of feasible solutions. In highway construction projects, standardization of cross-sectional dimensions of girders is done for different span lengths. This is done to minimize cost of form work. However, hardly any effort is done on overall cross section size to reduce the cost.

Optimization techniques such as utilization of Meta-heuristic algorithms change this trial-and-error process to a systematic, computer-based procedure that leads to an optimal or sub-optimal design solution without compromising structure's integrity. The success of the metaheuristic algorithm relies on imitation of biological phenomenon. As the nature tends to evolve towards favorable conditions, so does the metaheuristic algorithms with each iteration. The use of GA, inspired by Darwin's Theory, for optimization of prestressed girder bridge have been reported many times in literature. Aydin and Ayvaz [1] used GA for overall cost optimization of multiple bridges, taking number of spans, cross-sectional dimensions of prestressed girders, and the area of Prestressing Steel (PS) as design variables. The solution turned out to be 12.6% more economical than the actual project. Kaveh et al. [2] presented that Modified Colliding Bodies optimization (MCBO) algorithm performs better than CBO and PSO for cost-optimum design of post-tensioned box girder bridges by 0.4% to

2.2% for various span lengths and cross-sectional widths. Nour at. al. [3] demonstrated the optimization of fully and partially prestressed girder, with the help of methodology based on GA. A sensitivity analysis was also performed on numerical problems. Depth of beam was reduced by 39% by increasing concrete strength from 30MPa to 60MPa. Optimal design of cantilever retaining walls under seismic loads was done by Temur [4] using a hybrid TLBO algorithm. The solution was applied on three numerical problems and the results were compared with nine other, commonly used algorithms. For a population size of 50 and 100 runs, Hybrid TLBO yielded better results than its competitors. Munoz et. al. [5] successfully employed a novel, hybrid meta-heuristic approach to reduce CO_2 emission of a box girder steel-concrete composite bridge.

Ozturk et. al. [6] used Jaya Algorithm and TLBO for cost optimization of counterfort retaining wall. A total of seventeen variables and forty-six constraints were used. It was concluded in the research that TLBO performs better than Jaya Algorithm by almost 4%, however the latter converged more quickly towards the desired solution. Degertekin et. al. [7] employed a novel, parameter-free Jaya Algorithm for sizing and layout optimization of truss structure. Efficiency was demonstrated by applying algorithm to eight benchmark truss problems with largest subject truss having fifty-nine discreet variables and was found to yield better solutions than other methods. Standard deviation in optimized weight was very small after just twenty independent runs. Sami et. al. [8] developed a software program, employing Jaya Algorithm and ETABS. The software successfully optimized real life building structures comprising of shear walls and RC frames. Aydin et. al. [9] employed Jaya Algorithm for design optimization of steel truss, wherein prestressing steel and layout of truss beams were considered variables. It was concluded that shape variables (profile of prestressing steel and layout of truss beams) results in significant cost savings compared to size variables. As prestressed concrete I girder has not yet been designed by this algorithm, its efficiency for this specific problem is yet to be tested.

2 Problem Formulation

2.1 Design Variables and Constants.

Material properties of concrete and reinforcing steel, number of tendons, and tendon profile were considered as constant design parameters. Design variables included cross sectional geometry and number of strands per tendon. Design parameters and their ranges used in the optimization are given in the following Table 1.

Table 1: Design Parameters Ranges

Design Parameter	Variable	Units	Range
Top flange Width	b_1	mm	$500 \le b_1 \le 2000$
Bottom Flange Width	b_2	mm	$300 \le b_2 \le 1000$
Web Thickness	b_3	mm	$100 \le b_3 \le 500$
Total Depth of Girder	d_1	mm	$1500 \le d_1 \le 3500$
Top Flange Thickness	d_2	mm	$100 \le d_2 \le 400$
Top Flange Transition Thickness	d_3	mm	$100 \le d_3 \le 400$
Bottom Flange Transition Thickness	d_4	mm	$50 \le d_4 \le 400$
Bottom Flange Thickness	d_5	mm	$50 \le d_5 \le 700$
Strands per Tendon	s	No	$5 \le s \le 15$

2.2 Objective Function.

Material cost was taken as an objective function to minimize the cost of girder. The cost of formwork and construction were incorporated in the cost of concrete. The objective function is defined in the following Equation (2.1).

$$\text{Minimize } C_T = (C_C \times V_C) + (C_{PS} \times W_{PS}) + \text{Penalty} \quad (2.1)$$

Where the variables C_C, V_C, C_{PS}, and W_{PS} are defined as concrete cost, concrete volume, prestressing steel cost, and weight of the prestressing steel, respectively. A massive penalty of 10^8 was applied if any of the constraints were violated.

2.3 Design Constraints.

General design procedure and limitations followed in this study were according to the guidelines of AASHTO LRFD design manual (2017). Total number of constraints considered were 30.

The allowable stress in girder at top and bottom was defined as follows:

$$\sigma_C \le \sigma_W \le \sigma_T \tag{2.2}$$

$$\sigma_W = \frac{P_e}{A} \pm \frac{P_e e}{S} \pm \frac{M}{S} \tag{2.3}$$

Where σ_C is stress in compression, σ_W is allowable working stress, and σ_T is stress in tension. This constraint was satisfied for various loading stages of prestressed girder. Pe, e, A, S and M are Prestressing force, eccentricity, concrete area, section modulus, and working moments at a particular section, respectively. Allowable stresses were checked at three stages for two load combinations.

The ultimate flexural strength constraint was defined as

$$M_u \le \emptyset M_n \tag{2.4}$$

where M_u is factored moment and $\emptyset M_n$ is flexural strength of section. Flexural strength was calculated using AASHTO LRFD guidelines.

Also, enough reinforcement and prestressing should be provided to develop ultimate moment at least 1.2 times the critical moment.

$$1.2M_{cr} \le \emptyset M_n \tag{2.5}$$

The ultimate shear stress constraint is governed by following equation:

$$V_u = \emptyset(V_c + V_s + V_p) \tag{2.6}$$

where V_u is factored shear at a particular section, whereas V_c, V_s, and V_p are shear strength provided by concrete, shear reinforcement, and prestressing, respectively. Here, V_p was taken as zero as per Method 3 of AASHTO LRFD for calculation of shear capacity of the section.

The long-term deflection of the girder was also calculated and was limited by following expression:

$$\Delta \le {}^{L}/_{800} \tag{2.7}$$

where Δ is long term deflection and L is span length.

Geometrical constraints were also observed where minimum dimensions were governed by: i) clear cover, ii) clear spacing between tendons and reinforcements, iii) dimensions for bearing plate for jack for bottom flange and end block, and iv) span to top flange ratio to avoid failure of lateral bending at prestressing transfer stage.

3 Research Methodology

Most of evolutionary and swarm intelligence meta-heuristic algorithms require algorithm-specific parameters such as GA uses mutation, cross-over and selection operator, PSO uses social and cognitive parameters etc. The desired outcome of results is highly dependent on fine tuning of these parameters. Jaya Algorithm proposed by Rao [10] is single phase algorithm-specific-parameter-less algorithm. The user only chooses population size and number of iterations, hence, it is easy to operate than others.

Jaya Algorithm simply converges towards a desired solution while diverging away from undesired solutions. After definition of design variables (j), design constants, design constraints, population size (k), and number of iterations (i), initial set of design variables is generated randomly while obeying lower and upper bounds. Value of each variable is then updated stochastically using Eq. (3.1)

$$y^{i+1}_{j,k} = y^{i}_{j,k} + r^{i}_{j,1}\left(y^{i}_{j,kbest} - \left|y^{i}_{j,k}\right|\right) - r^{i}_{j,2}\left(y^{i}_{j,kworst} - \left|y^{i}_{j,k}\right|\right) \tag{3.1}$$

where i=1 to n and k=1 to m. $y^{i}_{j,k}$ is j^{th} variable of k^{th} solution at i^{th} iteration. $r^{i}_{j,1}$ and $r^{i}_{j,2}$ are random numbers in the range of (0,1). Their purpose is to ensure good diversification. $y^{i}_{j,kbest}$ and $y^{i}_{j,kworst}$ are the best and worst variables corresponding to k^{th} solution at any iteration "i". After updating the variables using Eq. (3.1), the values of variables are considered for next iteration only if the updated solution is better than the corresponding old solution. As the design of structures is bounded by constraints such as deflection and stress limitations, a heavy penalty of 10^8 is assigned to the solution upon the violation of constraints. A typical iterative cycle of Jaya algorithm is demonstrated in Figure 1.

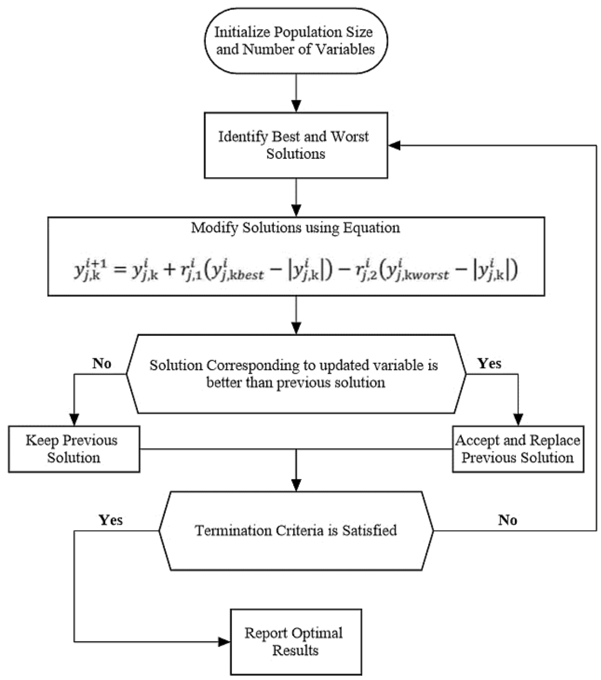

Figure 1: Flowchart of Optimization Process

4 Results and Discussion

A comparison of optimized girder section using Jaya Algorithm and numerical problem under similar design conditions was made to evaluate the algorithm's applicability. The optimization problem considered a forty meters long prestressed, post-tensioned interior "I" girder of bridge superstructure. The girder consisted of two (02) layers of six (06) tendons (03 in each layer) with an ultimate strength of 1862 MPa. The diameter of stress relieved strands used in this example was 12.7mm. Concrete flexural strength was kept constant at 41 MPa with a density of 24 kN/m^3. Unit weight of prestressing steel was kept at 0.775 kg/m. The cost of prestressing steel was taken as 224657.89 Rs/Ton, whereas the cost of concrete was kept at 15255.49 Rs/m^3 based on Composite Schedule Rate of National Highway Authority. Live loads were applied according to AASHTO LRFD and West Pakistan Code of Practice for Highway Bridges-1967. The applied dead loads and super imposed dead loads were computed within the program developed in MATLAB.

Program was run for 300 iterations 30 times for a population size of 50 and the result converged to almost the same solution after 200th iteration. The computational effort is minimum as 300 iterations were completed in few seconds each time. Comparison of total cost against number of iterations for first five trails are shown in the following Figure 2.

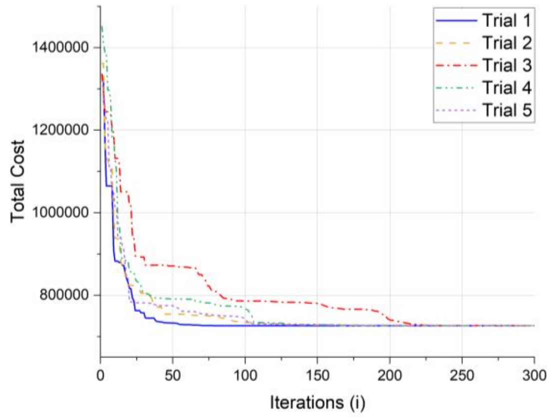

Figure 2:Convergence vs Iteration of Jaya Algorithm

The summarized results and comparison of sample and optimized section is provided in Table 2. The results consistently showed reduction of 29.81% in total cost of girder.

Table 2:Comparison of Result

Design Parameter	**Numerical Problem**	**Optimized Section**
Top flange Width (b_1) (mm)	1100	887
Bottom Flange Width (b_2) (mm)	750	615
Web Thickness (b_3) (mm)	190	163
Total Depth of Girder (d_1) (mm)	2500	3034
Top Flange Thickness (d_2) (mm)	170	100
Top Flange Transition Thickness (d_3) (mm)	225	100
Bottom Flange Thickness (d_5) (mm)	280	101
Bottom Flange Transition Thickness (d_4) (mm)	300	200
Strands per Tendon (s)	12	8
Cost	1,108,311 PKR	777,862 PKR

Mid-section of post-tensioned I girder designed by a typical hit-and-trial method and Jaya algorithm is shown in Figure 3a and Figure 3b, respectively. It can clearly be seen that prestressing contributes much more to the capacity of the section. Use of more tendons or strands per tendon will increase the capacity of section but it will be a costly solution.

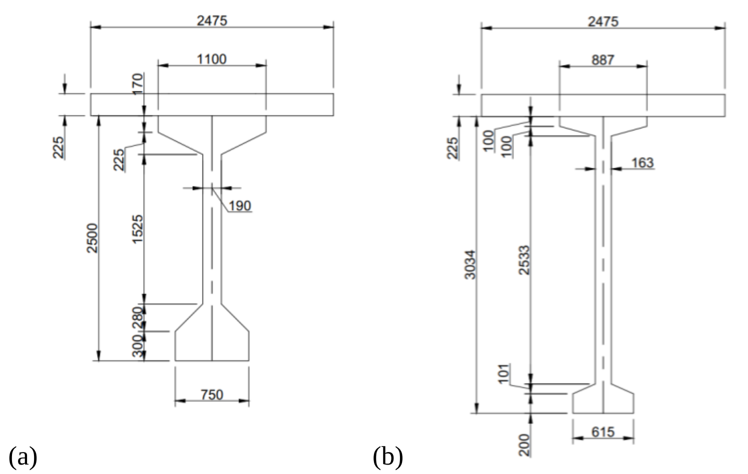

Figure 3: a. Section Designed by hit-and-trial method, b. Optimized Section

5 Practical Implementation

The program created using MATLAB is as easy to understand as other techniques in the industry such as finite element modeling software and excel sheets. The program can be improved to incorporate other features such as deck design, tendon profiling, and substructure design. It can also incorporate geometric features such as skew and curves of geometry to design more complex structures easily. For example, deciding between deck thickness and number of girders on curved geometry sometimes becomes a painfully tedious process. By providing few inputs to the software, it'll take less time to find the optimal option.

6 Conclusion

Following conclusions can be drawn from this study:

1 29.81% reduction in cost of girder mainly due to decrease in number of tendons.
2 Optimal design always results in a section with higher height to with ratio. A decrease in the number of tendons reduces the capacity of section which is accommodated by an increase in depth.
3 High convergence rate of Algorithm as it converged after approximately 200 iterations every time.
4 Time and computational efforts were minimum.

Use of similar algorithms in construction industry can help reduce the material and construction cost, and therefore, net embodied carbon. Since design stage of structural components are based on trial and error, simple yet robust working of Jaya Algorithm will result in saving time, computational efforts, and construction costs.

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